MATHEMATICS SPECIALIST UNITS 1&2

Section One:
Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section One: Calculator-free 35% (52 Marks)

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (7 marks)

Two matrices are $A = \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -5 \\ 10 & 5 \end{bmatrix}$. Determine

(a) $3A - 2B$. (2 marks)

(b) A^{-1}. (2 marks)

(c) $AB + B^2$. (3 marks)

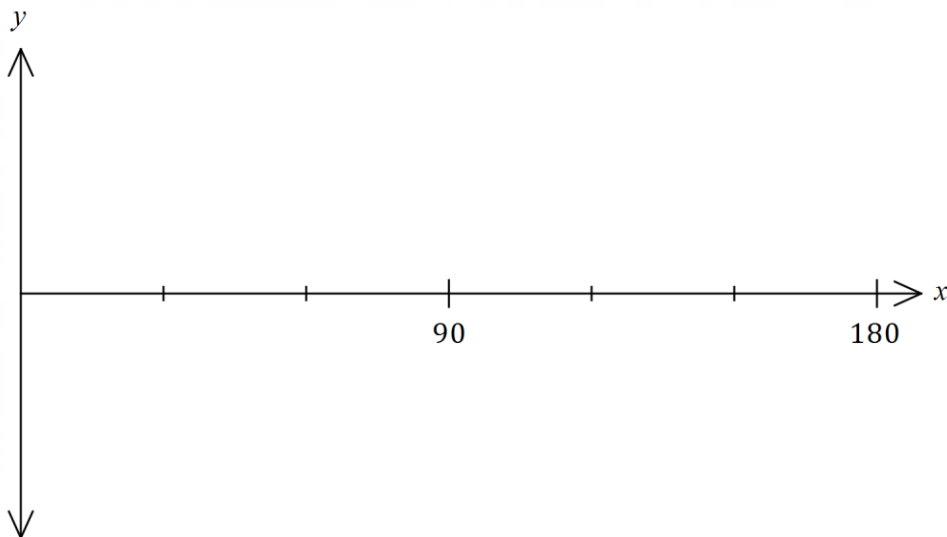**Question 2** **(6 marks)**

(a) State the exact value of $\cot 60°$. (1 mark)

(b) Given that $\sec \theta = \dfrac{3}{2}$ and $-90° \leq \theta \leq 0°$, state the exact value of $\operatorname{cosec} \theta$. (2 marks)

(c) Sketch the graph of $y = 2 \sec x$ for $0° \leq x \leq 180°$ on the axes below. (3 marks)

Question 3 **(8 marks)**

(a) Express $\left(\sqrt{5} + \sqrt{-5}\right)^2$ in the form $a + bi$ where $a, b \in \mathbb{R}$. (2 marks)

(b) Two complex numbers are $u = 8 + i$ and $v = 2 - i$. Calculate

   (i) $u \times v$. (1 mark)

   (ii) $u \div v$. (2 marks)

   (iii) $\text{Im}(2\bar{v} - iu)$. (3 marks)

Question 4 **(7 marks)**

(a) Use an angle sum identity to prove that $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$. (2 marks)

(b) Hence, or otherwise, prove that $\tan 3A = \dfrac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$. (3 marks)

(c) Solve $3 \tan A - \tan^3 A = 1 - 3 \tan^2 A$ for $0° < A < 30°$. (2 marks)

Question 5 **(5 marks)**

AC is a diameter of a circle centre O and point B lies on the circumference of the circle.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

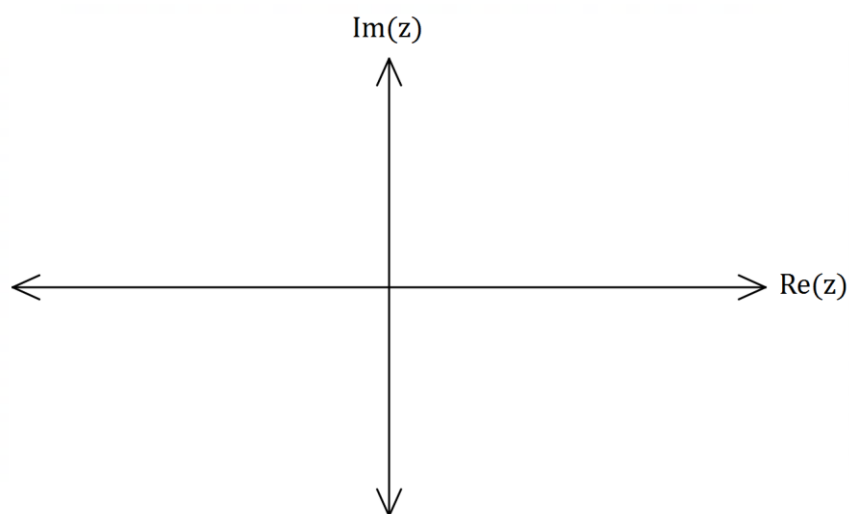Use a vector method to prove that $\angle ABC = 90°$.

Question 6 **(6 marks)**

(a) Determine the equation of the real quadratic $f(z)$ in the form $z^2 + az + b$ given that $f(5 - 3i) = 0$. (2 marks)

(b) Let $g(z) = z^2 - 8z + 17$.

(i) Determine z_1 and z_2, the complex roots of g. (2 marks)

(ii) Sketch and label z_1, z_2 and $w = z_1 + z_2$ in the complex plane below. (2 marks)

Question 7 **(7 marks)**

Use mathematical induction to prove the following proposition $P(n)$ for every integer $n \geq 0$.

$$P(n): 1 + 9 + 17 + 25 + \cdots + (8n + 1) = (n + 1)(4n + 1)$$

Question 8 **(6 marks)**

(a) Determine the vector projection of $2\mathbf{i} - \mathbf{j}$ on $-3\mathbf{i} + 4\mathbf{j}$. (2 marks)

(b) The vectors $a\mathbf{i} + \mathbf{j}$ and $4\mathbf{i} + b\mathbf{j}$ are perpendicular and the sum of their magnitudes is 10. Determine the values of the constants a and b. (4 marks)

End of questions

Supplementary page

Question number: _____